Chapter 2

THEORETICAL ANALYSIS OF BJT CLASS-F POWER AMPLIFIER

The concept of class-F power amplifier is based on the realization of open-circuit and short-circuit conditions for the higher harmonics at the transistor output[17]. The third harmonic peaking class-F is the widespread case of such power amplifiers[18-23]. The input impedance of output network represents the zero resistance for the second harmonic frequency and infinite value for the third harmonic frequency ideally.

The transistor output current and voltage impulses become half-wave truncated cosinusoidal and square-wave waveforms, respectively. This leads to the collector efficiency increasing due to the dissipated in transistor power decreasing. However, the above mentioned short-circuit and open-circuit requirements are not sufficient for generation of such waveforms. The phase relations between the harmonics of transistor output current should be taken into account. This problem was considered in details by Colantonio et al.[15,16]. The pure resistive load at the first harmonic frequency was assumed and truncated cosinusoid approximation was used for transistor output current waveform. It was shown[15,16] that the first and the third harmonics of transistor output current are out-of-phased for the conduction angles above $180°$. In this case, the first and the third harmonic Fourier coefficients have the opposite signs.

The effect of stretching of collector current impulse appears with the increased frequency becoming the transition one. This leads to the appreciable changes of harmonics' magnitudes and phases. Such stretching of transistor output current impulse was not taken into account in the Colantonio et al. analysis[15,16]. Therefore, it is interesting and useful to consider.

1. TRANSISTOR MODEL

The bipolar transistor behavior can be described satisfactorily by the charge control model within the substantial operating frequency band[24]. It allows to account the relations between the collector i_C and base i_B currents and the excess charge q of minority carriers in the base region and the charges accumulated in the nonlinear barrier emitter C'_e and collector C'_c barrier capacitances. The collector barrier capacitance C'_c is divided into two parts: the capacitance C'_{ca} of active part of collector junction and the capacitance C'_{cp} of passive part of collector junction. The C'_{ca} capacitance represents the part of displacement current flowing between the collector and emitter junction with voltage v_j across it. The C'_{cp} capacitance represents the part of displacement current flowing between the collector and base terminal. The relations for the coupling between the currents, base charges and voltages are the following[24]:

$$i_C = \frac{q}{\tau_T} + C'_{ca}\frac{d(v_{CE} - v_j)}{dt} + C'_{cp}\frac{d(v_{CE} - v_{BE})}{dt} \tag{2-1}$$

$$i_B = \frac{q}{\tau_\beta} + \frac{dq}{dt} - C'_{ca}\frac{d(v_{CE} - v_j)}{dt} - C'_{cp}\frac{d(v_{CE} - v_{BE})}{dt} + C'_e\frac{dv_j}{dt} \tag{2-2}$$

where τ_T is the average base carrier transit time; τ_β is the time constant with the value close to the average base minority carrier lifetime.

The minority carriers concentration gradient in the base region close to collector junction assumes to be proportional to the q [24]. Moreover, it is supposed that q is varying simultaneously with the excess minority carriers concentration in the base region close to the emitter junction. Latter is the exponential function of v_j, so the q can be expressed as follows:

$$q = q_{inv}[\exp(v_j / \varphi_T) - 1] \tag{2-3}$$

where $q_{inv} = I_{B0}\tau_\beta$; I_{B0} is the inverse base thermal current; $\varphi_T = kT / e$; k is the Boltzmann constant; T is the absolute temperature of emitter junction; e is the electron charge.

The junction voltage v_j and the v_{BE} voltage are coupled by the following equation:

$$v_j = v_{BE} - \left[i_B - C'_{cp} \frac{d(v_{BE} - v_{CE})}{dt} \right] r'_b \quad (2\text{-}4)$$

The τ_T value is defined through the transition frequency ω_T of common-emitter current transmission coefficient:

$$\tau_T = 1/\omega_T$$

while τ_β is expressed through the τ_T and dc current transmission coefficient β_{DC}:

$$\tau_\beta = \beta_{DC}\tau_T$$

The set of Eqs. (2-1) - (2-4) allows to define the i_C and i_B currents for the given transistor input and output voltages for the active or cut-off operating modes.

The equations for the static characteristics are obtained from the Eqs. (2-1) - (2-4) for the low operating frequencies case, where the displacement currents can be neglected[54]. After defining the $i_C = I_C$, $i_B = I_B$, $v_{BE} = V_{BE}$, the equations can be presented in the following form:

$$I_B = q/T_\beta = I_{B0}[\exp(v_j/\varphi_T) - 1] \quad (2\text{-}5)$$

$$I_C = \beta_{DC} I_B \quad (2\text{-}6)$$

$$V_{BE} = v_j + I_B r'_b$$

The nonlinear dependence Eq. (2-3) can be approximated by the piecewise linear one[24] in order to simplify the power amplifier operating analysis:

$$q = C_d (v_j - E')\Big|_{v_j > E'}, \quad (2\text{-}7)$$

where the C_d is the average diffusion capacitance of the effective part of the transistor active region. The differential capacitance $C'_d = dq/dv_j$ that can be found from Eq. (2-3) is proportional to the base stored charge: $C'_d = q/\varphi_T$ for the $q >> q_{inv}$. The C_d should be selected as $C_d = C_{d\,\max}/2 = q_{\max}/2\varphi_T$

for the piecewise linear approximation Eq. (2-7)[24]. The E' is the cut-off voltage that is boundary between the active and the cut-off modes. The representation of the base recombination current component dependence (2.5) on the v_j is as follow:

$$I_B = (1/r_\beta)(v_j - E')\Big|_{v_j > E'} \tag{2-8}$$

where

$$r_\beta = \tau_\beta / C_\partial$$

is the average resistance of parallel equivalent circuit of the base-to-emitter junction in the active region for the given C_d. As can be seen from the Eqs. (2-6) and (2-8), the dependence of $I_C(v_j)$ should be approximated as follow[24]:

$$I_C = S_j(v_j - E')\Big|_{v_j > E'} \tag{2-9}$$

where

$$S_j = \beta_{DC} / r_\beta$$

is the transconductance of the collector current on the base-to-emitter junction. The piecewise linear dependencies of $I_B(V_{BE})$, and $I_C(V_{BE})$ for the low frequencies can be obtained by substitution of Eq. (2-8) into (2-5) as follows[24]:

$$I_B = S_b(V_{BE} - E')\Big|_{V_{BE} > E'}$$

$$I_C = S(V_{BE} - E')\Big|_{V_{BE} > E'}$$

where

$$S_b = 1/(r'_b + r_\beta); \quad S = \beta_{DC} S_b = [r_\beta / (r'_b + r_\beta)] S_j .$$

The dependencies of nonlinear barrier capacitances C'_{ca}, C'_{cp}, C'_e on the voltages across them can be neglected in order to further simplification of the analysis[24]. The values of capacitances are assumed as the constants C_{ca}, C_{cp}, C_e, and averaged by the operating voltage diapason. Besides, the each of capacitances is usually small comparing with the diffusion one C_d.

Approximated charge control model can be represented by the equivalent circuit as shown in Fig. 2-1[24]. The switch K is closed for the $v_j > E'$, and is open for the $v_j < E'$. The model like this is also known as the Giacoletto model[25].

The bipolar junction transistor lag for the voltage source driving can be accounted in the model as follows. Let assume the cosinusoidal transistor input voltage:

$$V_{BE} = E_{BIAS} + V_{in} \cos \tau$$

The dependence of v_j on the $\tau = \omega_0 t$ have to be found in order to obtain the currents $i_C(\tau)$, and $i_B(\tau)$ according to the equivalent circuit shown in Fig. 2-1. The time constants of input circuit for the open-state transistor and close-state transistor are written as[24]:

$$\tau_S = \frac{r'_b \, r_\beta}{r'_b + r_\beta}(C_d + C_e)$$

$$\tau_e = r_b C_e$$

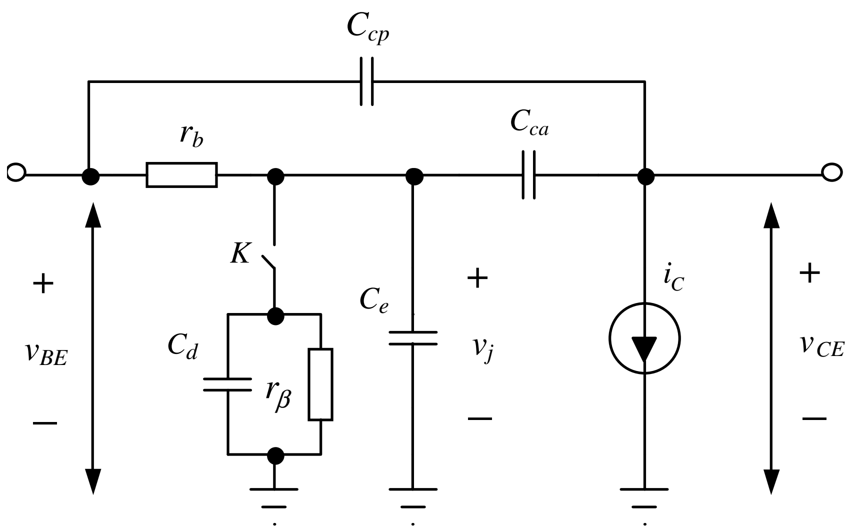

Figure 2-1. Equivalent circuit of BJT model.

The cut-off or close angle is defined as follows:

$$\cos\theta = -\frac{E_{BIAS} - E'}{V_{in}}$$

The θ is the low-frequency close angle, as it identifies the collector current cut-off for the $\omega_0 \to 0$[24]. The input circuit current transmission coefficient for the $v_j > E'$, and $\omega_0 \to 0$ is the following:

$$k_j = \frac{r_\beta}{r'_b + r_\beta}.$$

Therefore, the differential equations for the $v_j(\tau)$ of the open and the closed transistor states can be written as:

$$\begin{aligned} &\omega_0\tau_S\left(\frac{dv_j}{d\tau}\right) + v_j - E' = k_j V_{in}(\cos\tau - \cos\theta), \quad v_j > E' \\ &\omega_0\tau_e\left(\frac{dv_j}{d\tau}\right) + v_j - E' = V_{in}(\cos\tau - \cos\theta), \quad v_j < E' \end{aligned} \tag{2-10}$$

The assumption of $\tau_e = 0$ is valid for the $C_e << C_d$. In this case, the junction voltage $v_j(\tau)$ is equal to the input voltage in the cut-off region, and the transistor becomes open state at the $\tau = -\theta$. The solution of Eq. (2-10) for the open transistor with the initial condition $v_j(-\theta) = E'$ is the following:

$$\begin{aligned} v_j - E' = k_j V_{in} \Bigg\{ &\left[\frac{\cos(\tau + \varphi_S)}{\sqrt{1 + (\omega_0\tau_S)^2}} - \cos\theta \right] \\ &- \left[\frac{\cos(-\theta + \varphi_S)}{\sqrt{1 + (\omega_0\tau_S)^2}} - \cos\theta \right] \exp\left(-\frac{\tau + \theta}{\omega_0\tau_S} \right) \Bigg\}, \end{aligned} \tag{2.11}$$

where

$$\varphi_S = -\arctan\omega_0\tau_S$$

is the phase shift between the first harmonic of collector current and the input voltage.

The currents $i_C(\tau)$, and $i_B(\tau)$ can be found from the Eq. (2-11) for the $C_{ca} = C_{cp} = 0$ by taking into account the Eqs. (2-1), (2-2), and (2-9) as follows:

$$i_C(\tau) = S_j[v_j(\tau) - E']\Big|_{v_j > E'} \tag{2-12}$$

$$i_B(\tau) = \{[v_j(\tau) - E'] + \omega_0 C_d (dv_j(\tau)/d\tau)\}\Big|_{v_j > E'}$$

The harmonic analysis of the $i_C(\tau)$ is the subject of the next Section.

2. HARMONIC CONTENTS OF COLLECTOR CURRENT

The equivalent circuit of the BJT model[24] used for the spectral analysis is shown in Fig. 2-1. Several assumptions were made: BJT operates within the active and cut-off modes, without saturation; currents through C_{ca} and C_{cp} capacitances are much smaller than current of voltage controlled current source i_C, so they were neglected.

Therefore, the differential equation that describes the i_C according to the Eqs. (2-10) and (2-12) is as follows:

$$\omega_0 \tau_S \frac{di_C}{d(\omega_0 t)} + i_C = SV_{BE}(\cos\omega_0 t - \cos\theta), \tag{2-13}$$

Solution of equation (2-13) with $i_C(-\theta) = 0$ start condition gives the collector current on the interval from $-\theta$ to θ_1. The θ_1 is the transistor close angle. It can be defined from the transcendental equation as follows:

$$-\frac{\cos\theta}{\cos\varphi_S} + \cos(\theta_1 + \varphi_S) - \operatorname{tg}\varphi_S \sin(\theta - \varphi_S)\exp\left(\frac{\theta_1 + \theta}{\operatorname{tg}\varphi_S}\right) = 0\,. \tag{2-14}$$

The set of dependencies of θ_1 on θ obtained from Eq. (2-14) is shown in Fig. 2-2 with $\omega_0 \tau_S$ as parameter.

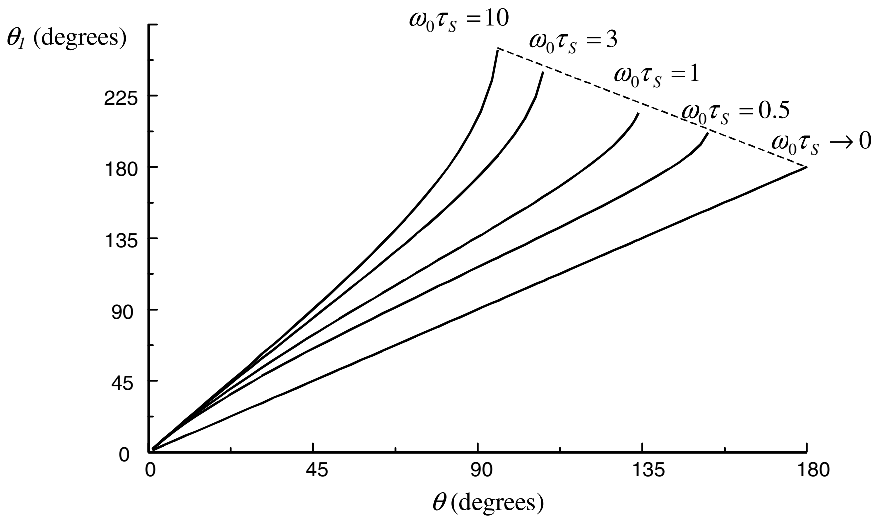

Figure 2-2. Set of dependencies of θ_1 on θ with $\omega_0\tau_S$ as parameter.

Thus, for known values θ_1, θ, and $\omega_0\tau_S$, the Fourier coefficients of collector current harmonics can be obtained, taking into account that i_C is equal to zero beyond the interval $(-\theta;\ \theta_1)$[26].

In order to find the dc component, both parts of Eq. (2.12) should be multiplied on $1/(2\pi)$ and integrated on $\omega_0 t$ within the $-\pi$ to π. However, the actual integration interval is from $-\theta$ to θ_1 due to zero current on intervals from $-\pi$ to $-\theta$, and θ_1 to π as follow:

$$\frac{\omega_0\tau_s}{2\pi}\int_{-\theta}^{\theta_1}\frac{di_C}{d(\omega_0 t)}d(\omega_0 t)+\frac{1}{2\pi}\int_{-\theta}^{\theta_1}i_C d(\omega_0 t)$$
$$=\frac{SV_{BE}}{2\pi}\int_{-\theta}^{\theta_1}(\cos\omega_0 t-\cos\theta)d(\omega_0 t). \qquad (2\text{-}15)$$

The first addend of the left part of Eq. (2-15) is equal to zero because of $i_C(-\theta)=i_C(\theta_1)=0$. The second addend presents the dc component of collector current:

$$a_0=\frac{1}{2\pi}\int_{-\theta}^{\theta_1}i_C d(\omega_0 t)\ .$$

Therefore, the following can be obtained from Eq. (2-15):

$$a_0 = \frac{SV_{BE}}{2\pi}\left(\sin\theta_1 + \sin\theta - (\theta_1 + \theta)\cos\theta\right).$$

The first harmonic cosinusoidal and sinusoidal Fourier components are the following, respectively:

$$a_1 = \frac{1}{\pi}\int_{-\theta}^{\theta_1} i_C \cos(\omega_0 t)d(\omega_0 t),$$

$$b_1 = \frac{1}{\pi}\int_{-\theta}^{\theta_1} i_C \sin(\omega_0 t)d(\omega_0 t).$$

In order to find them, both parts of Eq. (2-13) should be sequentially multiplied on $\cos(\omega_0 t)/\pi$, and $\sin(\omega_0 t)/\pi$, and integrated on $\omega_0 t$ from $-\theta$ to θ_1. As a result, a set of two algebraic equations can be achieved:

$$\begin{cases} -\omega_0\tau_S a_1 + b_1 = -\dfrac{SV_{BE}}{2\pi}B_1 \\ \omega_0\tau_S b_1 + a_1 = \dfrac{SV_{BE}}{2\pi}A_1 \end{cases}, \tag{2-16}$$

where

$$A_1 = \theta_1 + \cos\theta_1\sin\theta_1 + \theta - \cos\theta\sin\theta - 2\cos\theta\sin\theta_1,$$

$$B_1 = (\cos\theta - \cos\theta_1)^2.$$

From Eq. (2-16) the following can be obtained:

$$a_1 = \frac{SV_{BE}}{2\pi}\cdot\frac{\omega_0\tau_S B_1 + A_1}{(\omega_0\tau_S)^2 + 1},$$

$$b_1 = \frac{SV_{BE}}{2\pi}\cdot\frac{\omega_0\tau_S A_1 - B_1}{(\omega_0\tau_S)^2 + 1},$$

$$I_1 = \frac{SV_{BE}}{2\pi}\sqrt{\frac{{A_1}^2 + {B_1}^2}{(\omega_0\tau_S)^2 + 1}} ,$$

$$\varphi_{I_1} = \operatorname{arctg}\left(\frac{\omega_0\tau_S A_1 - B_1}{\omega_0\tau_S B_1 + A_1}\right).$$

The third harmonic cosinusoidal and sinusoidal Fourier components can be written as follows, correspondingly:

$$a_3 = \frac{1}{\pi}\int_{-\theta}^{\theta_1} i_C \cos(3\omega_0 t)d(\omega_0 t) ,$$

$$b_3 = \frac{1}{\pi}\int_{-\theta}^{\theta_1} i_C \sin(3\omega_0 t)d(\omega_0 t) .$$

In order to find a_3 and b_3, both parts of Eq. (2-13) should be sequentially multiplied on $\sin(3\omega_0 t)/\pi$, and $\cos(3\omega_0 t)/\pi$, and integrated on $\omega_0 t$ from $-\theta$ to θ_1. Then, the set of two algebraic equations can be obtained as follows:

$$\begin{cases} -3\omega_0\tau_S a_3 + b_3 = \dfrac{SV_{BE}}{6\pi}B_3 \\ 3\omega_0\tau_S b_3 + a_3 = \dfrac{SV_{BE}}{3\pi}A_3 \end{cases} , \tag{2-17}$$

where

$$A_3 = \sin\theta_1\cos^2\theta_1(3\cos\theta_1 - 4\cos\theta) + \cos\theta\left(\sin^3\theta + \sin\theta_1\right) ,$$

$$B_3 = 3\cos^2\theta_1 - 6\cos^4\theta_1 + 3\cos^2\theta \\ -2\cos^4\theta + 2\cos\theta\cos\theta_1\left(4\cos^2\theta_1 - 3\right) .$$

Therefore, the third harmonic Fourier coefficients can be found as follows:

$$a_3 = \frac{SV_{BE}}{6\pi} \cdot \frac{2A_3 - 3\omega_0 \tau_S B_3}{(3\omega_0 \tau_S)^2 + 1},$$

$$b_3 = \frac{SV_{BE}}{6\pi} \cdot \frac{6\omega_0 \tau_S A_3 + B_3}{(3\omega_0 \tau_S)^2 + 1},$$

$$I_3 = \frac{SV_{BE}}{6\pi} \sqrt{\frac{4A_3{}^2 + B_3{}^2}{(3\omega_0 \tau_S)^2 + 1}},$$

$$\varphi_{I_3} = \mathrm{arctg}\left(\frac{6\omega_0 \tau_S A_3 + B_3}{2A_3 - 3\omega_0 \tau_S B_3} \right).$$

3. CLASS F REALIZATION CONDITIONS

The Fourier coefficients of collector-emitter voltage v_{CE} are defined by appropriate Fourier coefficients of collector current i_C and input impedance of output network. The first and the third harmonic v_{CE} components can be written as follows, respectively:

$$\mathbf{U_1} = \mathbf{I_1 Z_1}, \ \mathbf{U_3} = \mathbf{I_3 Z_3}, \tag{2-18}$$

The output network should provide the matching with load at the fundamental frequency. Its input impedance $\mathbf{Z_1}$ is represented by resistance R_1. As the phase of $\mathbf{Z_1}$ is zero, the phase of $\mathbf{U_1}$ is equal to the phase of $\mathbf{I_1}$:

$$\varphi_{U_1} = \varphi_{I_1} \tag{2-19}$$

The phase of the third harmonic of collector-emitter voltage is as follows:

$$\varphi_{U_3} = \varphi_{I_3} + \varphi_{Z_3} \tag{2-20}$$

In order to achieve the proper waveform of collector voltage, it is necessary, that maximum of the fundamental frequency component and minimum of the third harmonic should be conterminous. This requirement is reduced to the simple out-of-phase requirement[15,16], if the phase of the first harmonic of collector voltage is equal to zero. But for non-zero phase of the first harmonic, this requirement has to be corrected. The first and the third harmonics can be represented as follows, respectively:

$$v_1(t) = V_1 \cdot \cos(\omega_0 t - \varphi_{V_1}) \,,$$

$$v_3(t) = V_3 \cdot \cos(3\omega_0 t - \varphi_{V_3}) \,.$$

The derivatives of these harmonics are the following:

$$v_1'(t) = -V_1 \cdot \omega_0 \sin(\omega_0 t - \varphi_{V_1}) \,, \qquad (2\text{-}21)$$

$$v_3'(t) = -V_3 \cdot 3\omega_0 \sin(3\omega_0 t - \varphi_{V_3}) \,. \qquad (2\text{-}22)$$

The phase angle for which $v_1(t)$ is in maximum should be determined. The obvious result φ_{V_1} can be achieved by equating the Eq. (2-21) to zero and solving:

$$\omega_0 t = \varphi_{V_1} \,. \qquad (2\text{-}23)$$

Equating the Eq. (2-22) to zero, substituting the Eq. (2-23), and solving give the phase angle of third harmonic φ_{V_3} for which $v_3(\varphi_{V_1}/\omega_0)$ is the minimum:

$$\varphi_{V_3} = 3\varphi_{V_1} + \pi \qquad (2\text{-}24)$$

Equation (2-24) defines the phase of the third harmonic, which is required for obtain target class-F collector voltage waveform. It reduces to out-of-phase requirement in case of $\varphi_{V_1} = 0$.

Using (2-19), φ_{V_3} can be written as follows:

$$\varphi_{V_3} = 3\varphi_{I_1} + \pi \,. \qquad (2\text{-}25)$$

The phase shift from $-\pi/2$ to $\pi/2$ can only be obtained by passive network. This gives the phase of input impedance of output network at the third harmonic within this region. Therefore, the phase of collector current third harmonic should match the following condition, according to Eqs. (2-20) and (2-21):

$$3\varphi_{I_1} + \pi/2 \le \varphi_{I_3} \le 3\varphi_{I_1} + 3\pi/2 \qquad (2.26)$$

The dependence of "allowed" conduction angles on $\omega_0\tau_S$ is shown in Fig. 2-3. The out-of-phase first and third harmonics can be obtained for these angles by adjustment the output network. One can see, that the "allowed" region is displaced to smaller conduction angles with frequency increasing. Therefore, the class-F operation with conduction angles below 90° becomes possible for quite high frequencies ($\omega_0 \ge 0.5 f_S$, where $f_S = 1/(2\pi\tau_S)$). However, the output power is decreased with decreasing the conduction angle that should be noted in design.

The set of dependencies of first harmonic collector current magnitude on conduction angle is shown in Fig. 2-4 for different values of $\omega_0\tau_S$. The magnitude is decreased noticeably with frequency growing for the same conduction angles. Therefore, an output power capability is decreased as well.

The relation between the third and the first harmonics of class-F amplifier transistor output voltage should be the following[27]:

$$V_3/V_1 = 1/6 .$$

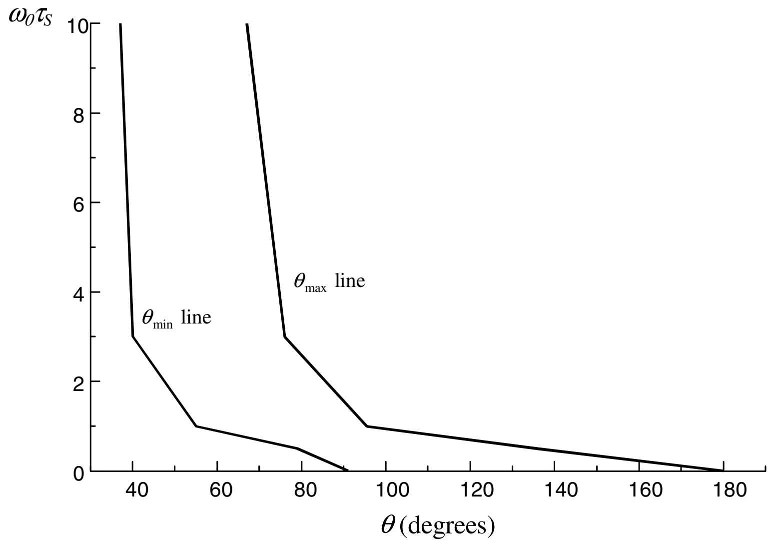

Figure 2-3. The dependence of "allowed" conduction angles on $\omega_0\tau_S$.

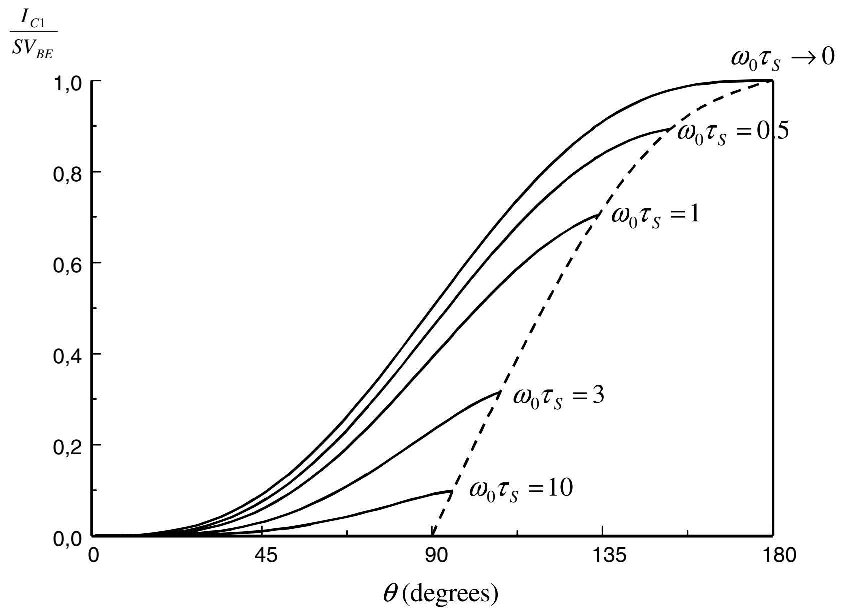

Figure 2-4. Set of dependencies of first harmonic collector current magnitude on conduction angle.

It gives the maximum efficiency and output power. Therefore, the required input impedance of output network at the third harmonic frequency can be found using the Eqs. (2-18) - (2-20) as follows:

$$Z_3 = \frac{I_1 R_1}{6I_3}, \tag{2.27}$$

$$\varphi_{Z_3} = 3\varphi_{I_1} + \pi - \varphi_{I_3}. \tag{2.28}$$

4. FIFTH-HARMONIC PEAKING ANALYSIS: VOLTAGE WAVEFORM PARAMETERS

In case of fifth-harmonic peaking class-F power amplifier, the transistor output voltage consists of the first, the third, and the fifth harmonics, and the dc-component as follows:

$$
\begin{aligned}
v_C &= E_C - V_{C1}\cos\omega_0 t - V_{C3}\cos 3\omega_0 t - V_{C3}\cos 5\omega_0 t \\
&= E_C - V_{C1}\left(\cos\omega_0 t + \frac{1}{\varepsilon_3}\cos 3\omega_0 t + \frac{1}{\varepsilon_5}\cos 5\omega_0 t\right)
\end{aligned}
\tag{2-29}
$$

where

$$
\varepsilon_3 = \frac{V_{C1}}{V_{C3}} = \frac{R_{\omega 0}}{R_{3\omega 0}} \cdot \frac{I_{C1}}{I_{C3}}, \tag{2-30}
$$

$$
\varepsilon_5 = \frac{V_{C1}}{V_{C5}} = \frac{R_{\omega 0}}{R_{5\omega 0}} \cdot \frac{I_{C1}}{I_{C5}}. \tag{2-31}
$$

The value of ε_3 should be negative, and the value of ε_5 should be positive. In this case, the voltage waveform can be close to maximally flat[14], or maximum output power[27].

The first harmonic magnitude is greater than half of the signal swing for the signal with out-of-phase the first and the third harmonics. It allows increasing the output power without overload. The growing of the first harmonic magnitude can be expressed as:

$$
V_{C1,F} = \gamma(\varepsilon_3,\, \varepsilon_5) V_{C1,\sin}, \tag{2-32}
$$

where the $\gamma(\varepsilon_3, \varepsilon_5)$ is the class-F first harmonic growing coefficient defined by the relation of the first harmonic magnitude to the magnitude of overall voltage swing as follows:

$$
\gamma(\varepsilon_3, \varepsilon_5) = \frac{V_{C1}}{|v(\theta_m) - E_C|} = \frac{1}{\cos\theta_m + 1/\varepsilon_3 \cos 3\theta_m + 1/\varepsilon_5 \cos 5\theta_m}.
$$

Here, the θ_m is the point of minimal or maximal value of the voltage impulse v_C, which can be determined from (2-29).

In order to find θ_m, the derivative of v_C on $\omega_0 t = \theta$ should be equated to zero. Then, the obtained equation should be solved. The derivative of v_C can be written as follows:

$$
v_C' = -V_{C1}(-\sin\theta - 3/\varepsilon_3 \sin 3\theta - 5/\varepsilon_5 \sin 5\theta). \tag{2-33}
$$

By equating the right part of Eq. (2-33) to zero as follows:

$$-V_{C1}\left(-\sin\theta - 3/\varepsilon_3 \sin 3\theta - 5/\varepsilon_5 \sin 5\theta\right) = 0\,, \tag{2-34}$$

the points of possible extremums of v_C within the $0 \le \theta \le \pi$ region can be found:

$$\theta_{m1} = 0\,, \tag{2-35}$$

$$\theta_{m2} = \arccos\sqrt{\frac{3}{8} - \frac{3\varepsilon_5}{40\varepsilon_3} + \frac{\sqrt{125\varepsilon_3^2 - 30\varepsilon_3\varepsilon_5 - 20\varepsilon_3^2\varepsilon_5 + 9\varepsilon_5^2}}{40\varepsilon_3}}\,. \tag{2-36}$$

The following expressions for the $\gamma_1(\varepsilon_3, \varepsilon_5)$ and $\gamma_2(\varepsilon_3, \varepsilon_5)$ functions are correspond to the θ_{m1} and θ_{m2}, respectively:

$$\begin{aligned}\gamma_1(\varepsilon_3,\varepsilon_5) &= \frac{1}{\cos\theta_{m1} + 1/\varepsilon_3 \cos 3\theta_{m1} + 1/\varepsilon_5 \cos 5\theta_{m1}} \\ &= \frac{\varepsilon_3\varepsilon_5}{\varepsilon_3 + \varepsilon_5 + \varepsilon_3\varepsilon_5},\end{aligned} \tag{2-37}$$

$$\begin{aligned}\gamma_2(\varepsilon_3,\varepsilon_5) &= \frac{1}{\cos\theta_{m2} + 1/\varepsilon_3 \cos 3\theta_{m2} + 1/\varepsilon_5 \cos 5\theta_{m2}} \\ &= \frac{50\sqrt{10}\varepsilon_3^2\varepsilon_5}{\sqrt{\left(15\varepsilon_3 - 3\varepsilon_5 + \sqrt{A_\varepsilon}\right)/\varepsilon_3}} \\ &\times \frac{1}{25\varepsilon_3^2 - 30\varepsilon_3\varepsilon_5 + 20\varepsilon_3^2\varepsilon_5 - 3\varepsilon_5 + (\varepsilon_5 - 5\varepsilon_3)\sqrt{A_\varepsilon}},\end{aligned} \tag{2-38}$$

where

$$A_\varepsilon = 125\varepsilon_3^2 - 10\varepsilon_3\varepsilon_5(3 + 2\varepsilon_3) + 9\varepsilon_5^2\,. \tag{2-39}$$

As it were a priory supposed, value of ε_3 is negative, and value of ε_5 is positive. Therefore, the function $\gamma_2(\varepsilon_3, \varepsilon_5)$ is real for the nonnegative values of A_ε only. The solution of the following equation:

$$A_\varepsilon = 0$$

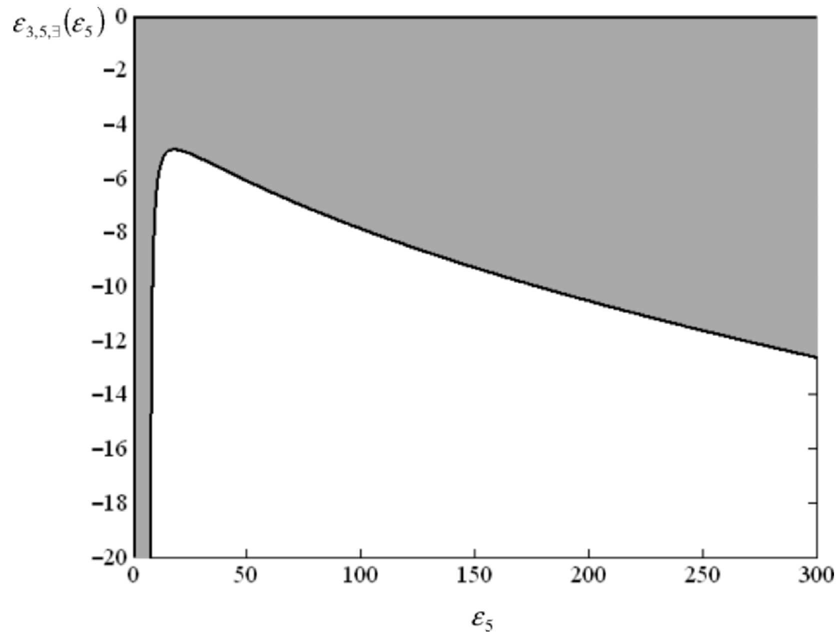

Figure 2-5. Curve $\varepsilon_{3,5,\exists}(\varepsilon_5)$, that restricts the range of definition of function $\gamma_2(\varepsilon_3, \varepsilon_5)$.

gives the following expression for the curve, that restricts the range of definition of function $\gamma_2(\varepsilon_3, \varepsilon_5)$:

$$\varepsilon_{3,5,\exists}(\varepsilon_5) = -\frac{3\left(5\varepsilon_5 + 2\sqrt{5}\varepsilon_5\sqrt{\varepsilon_5 - 5}\right)}{5(4\varepsilon_5 - 25)} \tag{2-40}$$

The function $\varepsilon_{3,5,\exists}(\varepsilon_5)$ is shown in Fig. 2-5. The range of definition of function $\gamma_2(\varepsilon_3, \varepsilon_5)$ is highlighted as well.

There is only one solution of Eq. (2-34) for the points $(\varepsilon_3, \varepsilon_5)$, which are beyond the range of definition of function $\gamma_2(\varepsilon_3, \varepsilon_5)$. This solution is defined by Eq. (2-35).

The functions $\gamma_1(\varepsilon_3, \varepsilon_5)$ and $\gamma_2(\varepsilon_3, \varepsilon_5)$ are shown in Figs. (2-6) and (2-7), respectively.

The functions $\gamma_1(\varepsilon_3, \varepsilon_5)$ and $\gamma_2(\varepsilon_3, \varepsilon_5)$ are equal to one another along two curves. These curves of contingence $\varepsilon_{3,5,MF}(\varepsilon_5)$ and $\varepsilon_{3,5,\max}(\varepsilon_5)$ can be found by equating the right parts of (2-37) and (2-38). Then, obtained equation should be solved for ε_3. The equating of (2-37) and (2-38) gives the following:

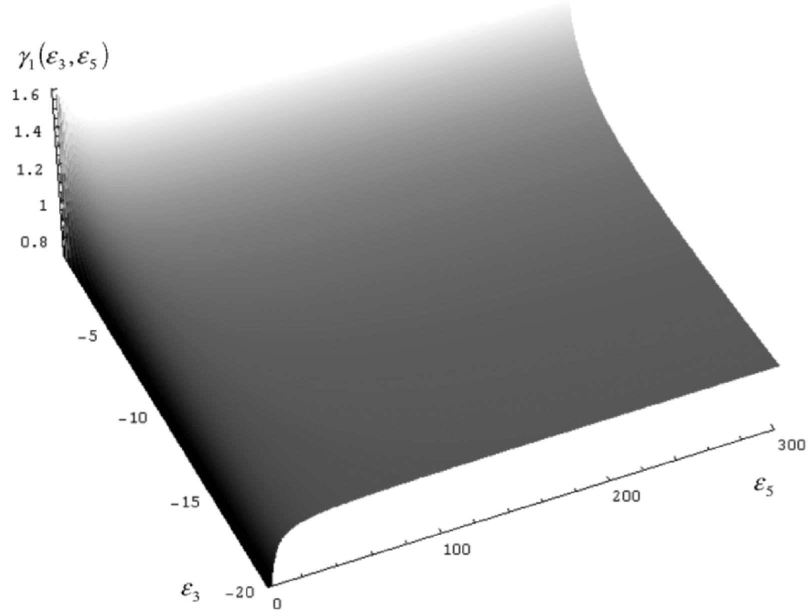

Figure 2-6. Function $\gamma_1(\varepsilon_3, \varepsilon_5)$.

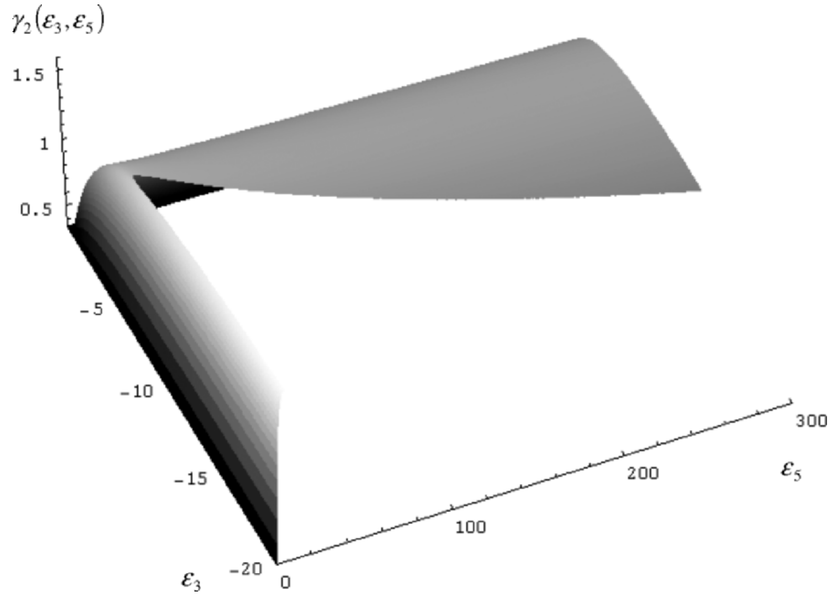

Figure 2-7. Function $\gamma_2(\varepsilon_3,\varepsilon_5)$.

$$\begin{aligned}&\frac{\varepsilon_3\varepsilon_5}{\varepsilon_3+\varepsilon_5+\varepsilon_3\varepsilon_5}\\&=\frac{50\sqrt{10}\varepsilon_3^2\varepsilon_5}{\sqrt{\left(15\varepsilon_3-3\varepsilon_5+\sqrt{A_\varepsilon}\right)/\varepsilon_3}}\\&\times\frac{1}{25\varepsilon_3^2-30\varepsilon_3\varepsilon_5+20\varepsilon_3^2\varepsilon_5-3\varepsilon_5+\left(\varepsilon_5-5\varepsilon_3\right)\sqrt{A_\varepsilon}}.\end{aligned} \tag{2-41}$$

The following expressions for the curves $\varepsilon_{3,5,MF}(\varepsilon_5)$ and $\varepsilon_{3,5,\max}(\varepsilon_5)$ can be found by solving the Eq. (2-41):

$$\varepsilon_{3,5,MF}(\varepsilon_5)=-\frac{9\varepsilon_5}{\varepsilon_5+25}, \tag{2-42}$$

$$\begin{aligned}\varepsilon_{3,5,\max}(\varepsilon_5)=&-\frac{A_{\varepsilon12}}{4A_{\varepsilon10}}-\frac{\sqrt{A_{\varepsilon2}+A_{\varepsilon3}}}{2}\\&-\frac{1}{2}\sqrt{2A_{\varepsilon2}-A_{\varepsilon3}-\frac{A_{\varepsilon1}}{4\sqrt{A_{\varepsilon2}+A_{\varepsilon3}}}},\end{aligned} \tag{2-43}$$

where

$$A_{\varepsilon1}=-\frac{A_{\varepsilon12}^3}{A_{\varepsilon10}^3}+\frac{4A_{\varepsilon11}A_{\varepsilon12}}{A_{\varepsilon10}^2}-\frac{8A_{\varepsilon7}}{A_{\varepsilon10}},$$

$$A_{\varepsilon2}=\frac{A_{\varepsilon12}^2}{4A_{\varepsilon10}^2}-\frac{2A_{\varepsilon11}}{3A_{\varepsilon10}},$$

$$A_{\varepsilon3}=\frac{\sqrt[3]{2}A_{\varepsilon6}}{2A_{\varepsilon10}\sqrt[3]{A_{\varepsilon4}}}+\frac{\sqrt[3]{A_{\varepsilon4}}}{3\sqrt[3]{2}A_{\varepsilon10}},$$

$$\begin{aligned}A_{\varepsilon4}=&2A_{\varepsilon11}^3+A_{\varepsilon8}-9A_{\varepsilon7}A_{\varepsilon11}A_{12}+27A_{\varepsilon10}A_{\varepsilon7}^2+\\&+27A_{\varepsilon12}^2A_{\varepsilon5}-72A_{\varepsilon5}A_{\varepsilon10}A_{\varepsilon11},\end{aligned}$$

$$A_{\varepsilon 5} = \varepsilon_5^2 + \varepsilon_5^3,$$

$$A_{\varepsilon 6} = 625 + 2000\varepsilon_5 + 2130\varepsilon_5^2 + 1064\varepsilon_5^3 + 256\varepsilon_5^4,$$

$$A_{\varepsilon 7} = -6\varepsilon_5 - 5\varepsilon_5^2,$$

$$A_{\varepsilon 8} = \sqrt{A_{\varepsilon 9}},$$

$$\begin{aligned} A_{\varepsilon 9} = {} & 3375000000\varepsilon_5^2 + 24300000000\varepsilon_5^3 + 78306750000\varepsilon_5^4 \\ & + 147776400000\varepsilon_5^5 + 179460724500\varepsilon_5^6 + 145015498800\varepsilon_5^7 \\ & + 77734274481\varepsilon_5^8 + 26570467584\varepsilon_5^9 + 5239406592\varepsilon_5^{10} \\ & + 452984832\varepsilon_5^{11}, \end{aligned}$$

$$A_{\varepsilon 10} = 16\varepsilon_5 + 25,$$

$$A_{\varepsilon 11} = 25 + 13\varepsilon_5 - 8\varepsilon_5^2,$$

$$A_{\varepsilon 12} = 75 + 52\varepsilon_5.$$

The functions $\varepsilon_{3,5,MF}(\varepsilon_5)$ and $\varepsilon_{3,5,\max}(\varepsilon_5)$, which are defined by Eqs. (2-42) and (2-43), respectively, are shown in Fig. 2-8. The function $\varepsilon_{3,5,\exists}(\varepsilon_5)$, which is defined by Eq. (2-30), is depicted as well.

As can be seen from Fig. 2-8, the curves $\varepsilon_{3,5,MF}(\varepsilon_5)$, $\varepsilon_{3,5,\max}(\varepsilon_5)$, and $\varepsilon_{3,5,\exists}(\varepsilon_5)$ are devide the range of definition of function $\gamma(\varepsilon_3, \varepsilon_5)$ into the five regions 1 - 5. There is own type of the waveform of voltage v_C for each of these regions.

The waveform of voltage v_C, that is appropriate to $\varepsilon_3 = -3$ and $\varepsilon_5 = 20$ from the region 1, is shown in Fig. 2-9.

As can be seen from Fig. 2-9, the waveform of voltage v_C has two lobes. This waveform looks like third-harmonic peaking one, that was considered in details in the Chapter 1.

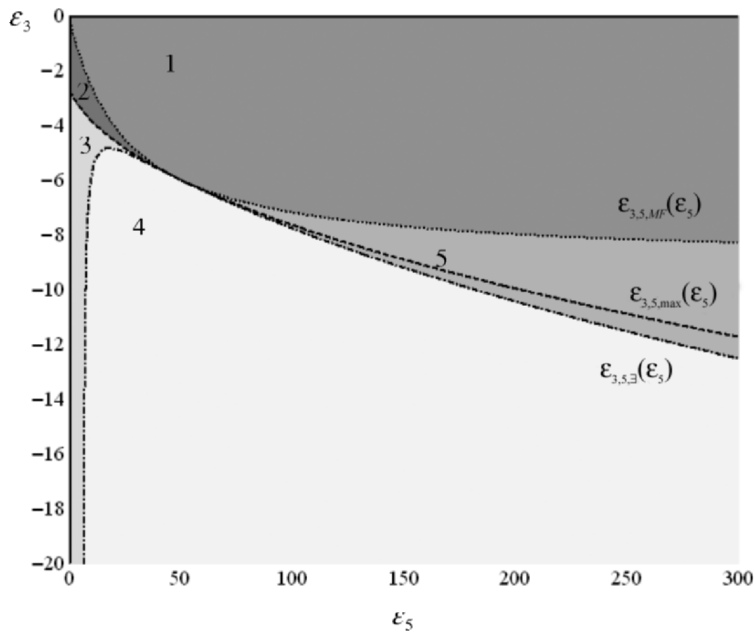

Figure 2-8. Functions $\varepsilon_{3,5,MF}\,(\varepsilon_5)$, $\varepsilon_{3,5,\max}\,(\varepsilon_5)$, and $\varepsilon_{3,5,\exists}(\varepsilon_5)$.

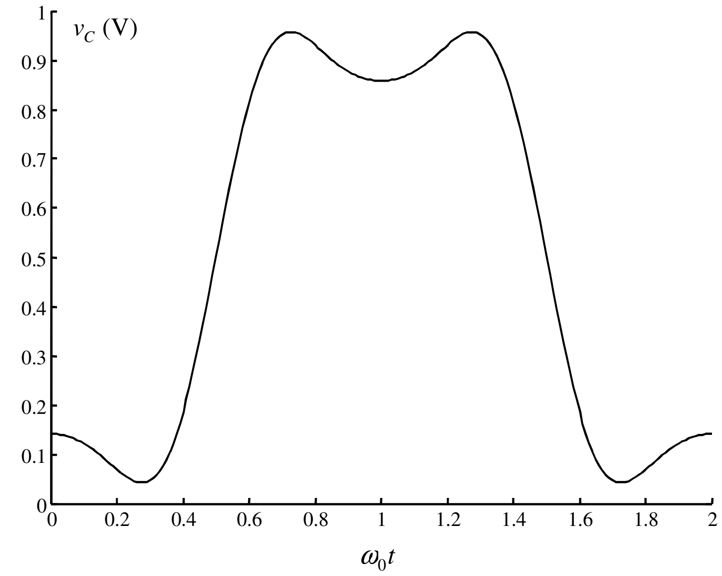

Figure 2-9. The waveform of voltage v_C, that is appropriate to $\varepsilon_3 = -3$ and $\varepsilon_5 = 20$ from the region 1.

The function $\gamma(\varepsilon_3, \varepsilon_5)$ is the same as function $\gamma_2(\varepsilon_3, \varepsilon_5)$ within the region 1:

$$\gamma(\varepsilon_3,\varepsilon_5)=\gamma_2(\varepsilon_3,\varepsilon_5)=\frac{50\sqrt{10}\varepsilon_3^2\varepsilon_5}{\sqrt{\left(15\varepsilon_3-3\varepsilon_5+\sqrt{A_\varepsilon}\right)/\varepsilon_3}} \times\frac{1}{25\varepsilon_3^2-30\varepsilon_3\varepsilon_5+20\varepsilon_3^2\varepsilon_5-3\varepsilon_5+(\varepsilon_5-5\varepsilon_3)\sqrt{A_\varepsilon}}, \text{ for region 1.} \tag{2-44}$$

The waveform of voltage v_C, that is appropriate to $\varepsilon_3 = -3.5$ and $\varepsilon_5 = 10$ from the region 2, is shown in Fig. 2-10.

As can be seen from Fig. 2-10, the waveform of voltage v_C has two simmetrical side lobes and one central. Besides, the central lobe is less in size than side lobes.

The function $\gamma(\varepsilon_3, \varepsilon_5)$ is the same as function $\gamma_2(\varepsilon_3, \varepsilon_5)$ within the region 2 as well:

$$\gamma(\varepsilon_3,\varepsilon_5)=\gamma_2(\varepsilon_3,\varepsilon_5)=\frac{50\sqrt{10}\varepsilon_3^2\varepsilon_5}{\sqrt{\left(15\varepsilon_3-3\varepsilon_5+\sqrt{A_\varepsilon}\right)/\varepsilon_3}} \times\frac{1}{25\varepsilon_3^2-30\varepsilon_3\varepsilon_5+20\varepsilon_3^2\varepsilon_5-3\varepsilon_5+(\varepsilon_5-5\varepsilon_3)\sqrt{A_\varepsilon}}, \text{ for region 2.} \tag{2-45}$$

The waveform of voltage v_C, that is appropriate to $\varepsilon_3 = -5$ and $\varepsilon_5 = 10$ from the region 3, is shown in Fig. 2-11. As can be seen from Fig. 2-11, the waveform of voltage v_C has two simmetrical side lobes and one central as well. However, the central lobe is greater in size than side lobes.

The function $\gamma(\varepsilon_3, \varepsilon_5)$ is the same as function $\gamma_1(\varepsilon_3, \varepsilon_5)$ within the region 3:

$$\gamma(\varepsilon_3,\varepsilon_5)=\gamma_1(\varepsilon_3,\varepsilon_5)=\frac{\varepsilon_3\varepsilon_5}{\varepsilon_3+\varepsilon_5+\varepsilon_3\varepsilon_5}, \text{ for region 3.} \tag{2-46}$$

The waveform of voltage v_C, that is appropriate to $\varepsilon_3 = -10$ and $\varepsilon_5 = 25$ from the region 4, is shown in Fig. 2-12. In this case, the waveform has the only central lobe, and the function $\gamma(\varepsilon_3, \varepsilon_5)$ is the same as $\gamma_1(\varepsilon_3, \varepsilon_5)$:

$$\gamma(\varepsilon_3,\varepsilon_5)=\gamma_1(\varepsilon_3,\varepsilon_5)=\frac{\varepsilon_3\varepsilon_5}{\varepsilon_3+\varepsilon_5+\varepsilon_3\varepsilon_5}, \text{for region 4.} \tag{2-47}$$

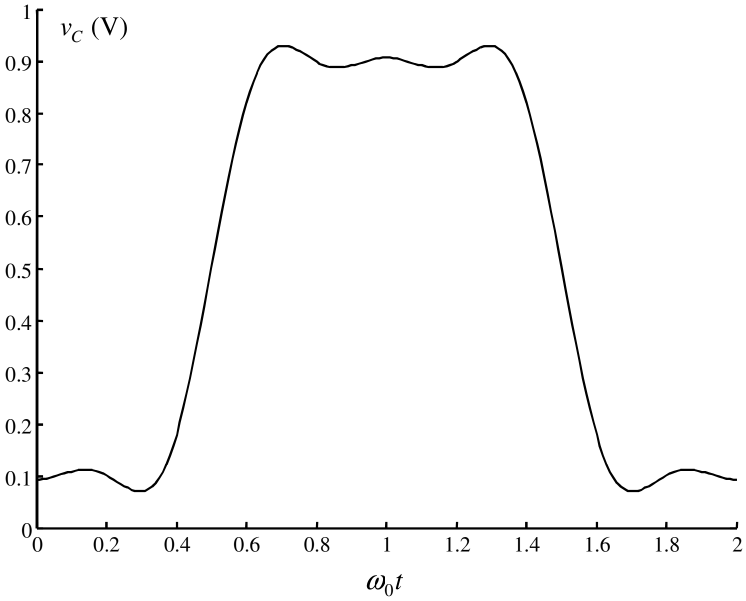

Figure 2-10. The waveform of voltage v_C, that is appropriate to $\varepsilon_3 = -3.5$ and $\varepsilon_5 = 10$ from the region 2.

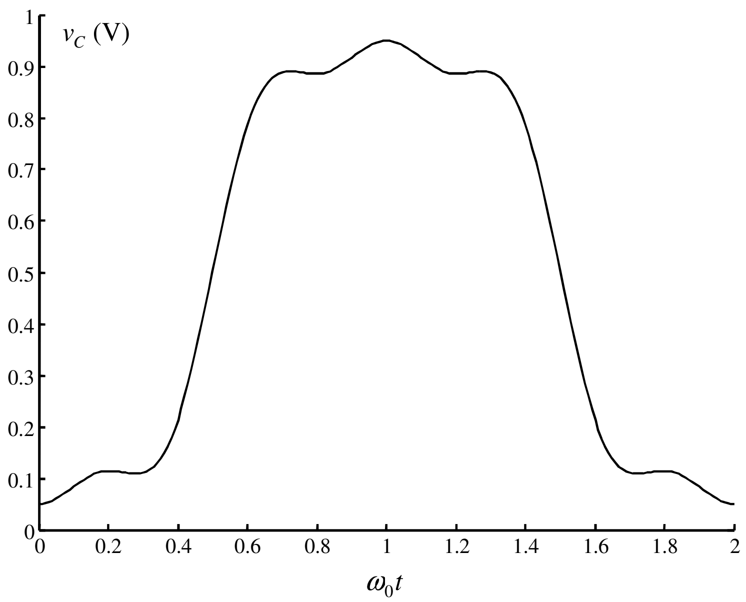

Figure 2-11. The waveform of voltage v_C, that is appropriate to $\varepsilon_3 = -5$ and $\varepsilon_5 = 10$ from the region 3.

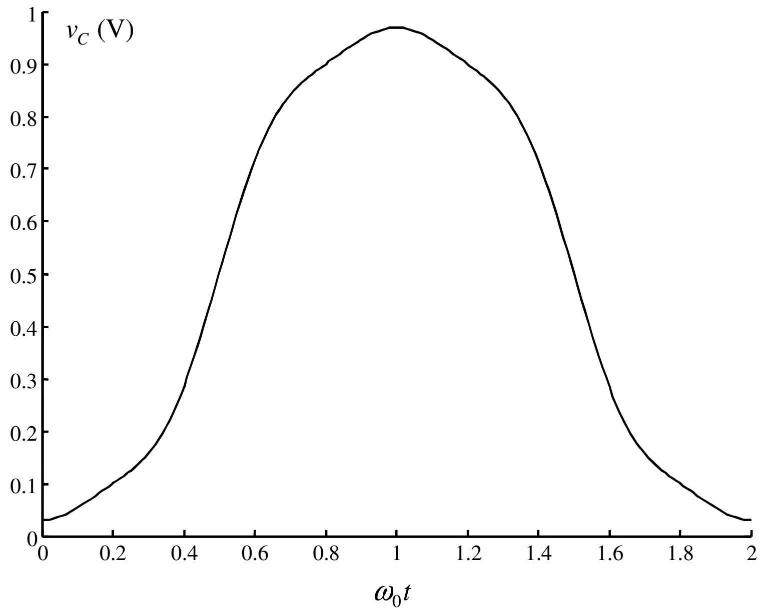

Figure 2-12. The waveform of voltage v_C, that is appropriate to $\varepsilon_3 = -10$ and $\varepsilon_5 = 25$ from the region 4.

The waveform of voltage v_C, that is appropriate to $\varepsilon_3 = -7.5$ and $\varepsilon_5 = 100$ from the region 5, is shown in Fig. 2-13. The waveform of voltage v_C has two side lobes and one central. However, the lobes are close to one another, and are of almost similar values. The function $\gamma(\varepsilon_3, \varepsilon_5)$ is the same as function $\gamma_1(\varepsilon_3, \varepsilon_5)$ within the region 5:

$$\gamma(\varepsilon_3, \varepsilon_5) = \gamma_1(\varepsilon_3, \varepsilon_5) = \frac{\varepsilon_3 \varepsilon_5}{\varepsilon_3 + \varepsilon_5 + \varepsilon_3 \varepsilon_5}, \quad \text{for region 5.} \tag{2-48}$$

The absciss $\varepsilon_{5,MF}$ of contingence point of $\varepsilon_{3,5,MF}(\varepsilon_5)$ and $\varepsilon_{3,5,\max}(\varepsilon_5)$ curves can be determined by equating the right parts of Eqs. (2-30), (2-32), and (2-33). In order to simplify the analysis, Eqs. (2-30) and (2-32) were selected as follows:

$$-\frac{3\left(5\varepsilon_{5,MF} + 2\sqrt{5}\varepsilon_{5,MF}\sqrt{\varepsilon_{5,MF} - 5}\right)}{5\left(4\varepsilon_{5,MF} - 25\right)} = -\frac{9\varepsilon_{5,MF}}{\varepsilon_{5,MF} + 25},$$

$$\varepsilon_{5,MF} = 50. \tag{2-49}$$

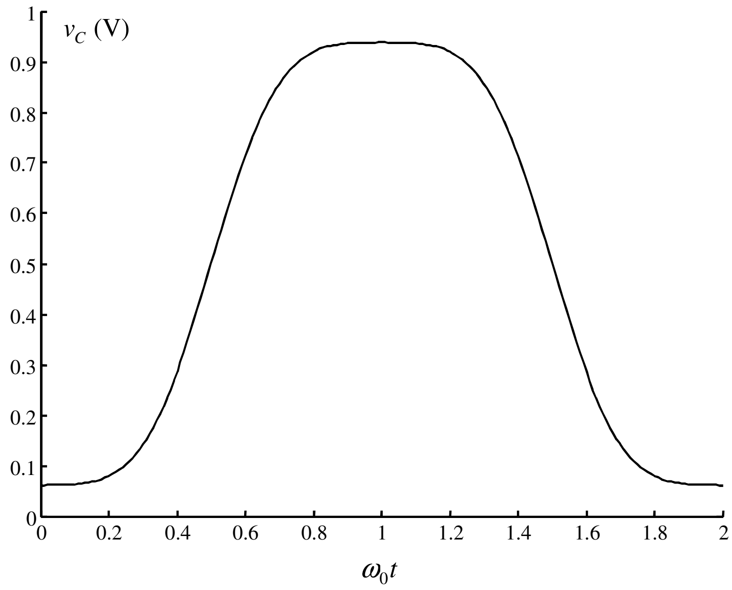

Figure 2-13. The waveform of voltage v_C, that is appropriate to $\varepsilon_3 = -7.5$ and $\varepsilon_5 = 100$ from the region 5.

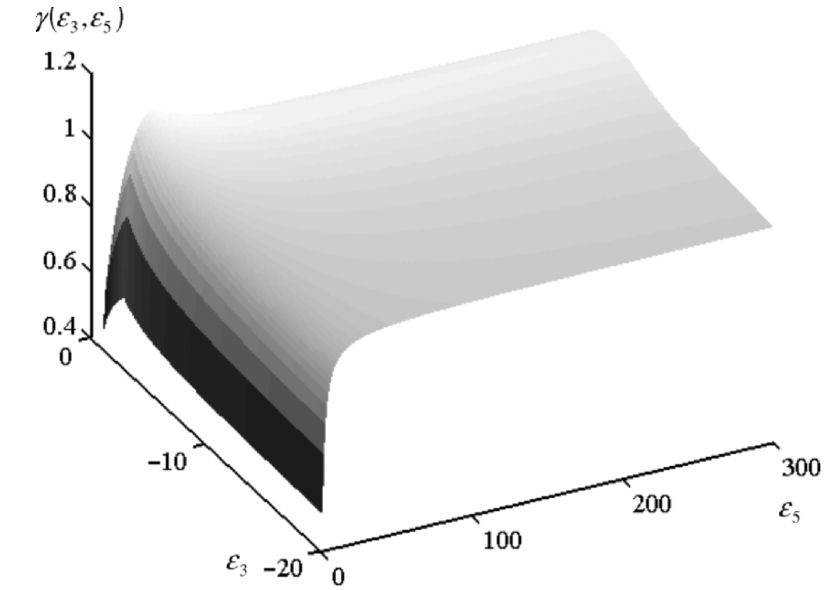

Figure 2-14. Function $\gamma(\varepsilon_3, \varepsilon_5)$.

The following ordinate is appropriate to the absciss $\varepsilon_{5,MF}$:

$$\varepsilon_{3,5,MF}\left(\varepsilon_{5,MF}\right)=\varepsilon_{3,5,\exists}\left(\varepsilon_{5,MF}\right)=\varepsilon_{3,5,\max}\left(\varepsilon_{5,MF}\right)=-6\,. \tag{2-50}$$

Therefore, the function $\gamma(\varepsilon_3,\varepsilon_5)$ for $\varepsilon_3<0$ and $\varepsilon_5>0$ values can be defined by the following set according to the Eqs. (2-44) − (2-48), taking into account Eqs. (2-49) and (2-50):

$$\gamma(\varepsilon_3,\varepsilon_5)=\begin{cases}\dfrac{50\sqrt{10}\varepsilon_3^2\varepsilon_5}{\sqrt{\left(15\varepsilon_3-3\varepsilon_5+\sqrt{A_\varepsilon}\right)/\varepsilon_3}}\\ \times\dfrac{1}{25\varepsilon_3^2-30\varepsilon_3\varepsilon_5+20\varepsilon_3^2\varepsilon_5-3\varepsilon_5+\left(\varepsilon_5-5\varepsilon_3\right)\sqrt{A_\varepsilon}},\\ \text{for } \varepsilon_5<=\varepsilon_{5,MF} \text{ and } \varepsilon_3>=\varepsilon_{3,5,\max}\left(\varepsilon_5\right),\\ \dfrac{50\sqrt{10}\varepsilon_3^2\varepsilon_5}{\sqrt{\left(15\varepsilon_3-3\varepsilon_5+\sqrt{A_\varepsilon}\right)/\varepsilon_3}}\\ \times\dfrac{1}{25\varepsilon_3^2-30\varepsilon_3\varepsilon_5+20\varepsilon_3^2\varepsilon_5-3\varepsilon_5+\left(\varepsilon_5-5\varepsilon_3\right)\sqrt{A_\varepsilon}},\\ \text{for } \varepsilon_5>\varepsilon_{5,MF} \text{ and } \varepsilon_3>=\varepsilon_{3,5,MF}\left(\varepsilon_5\right),\\ \dfrac{\varepsilon_3\varepsilon_5}{\varepsilon_3+\varepsilon_5+\varepsilon_3\varepsilon_5},\\ \text{for } \varepsilon_5<=\varepsilon_{5,MF} \text{ and } \varepsilon_3<\varepsilon_{3,5,\max}\left(\varepsilon_5\right),\\ \dfrac{\varepsilon_3\varepsilon_5}{\varepsilon_3+\varepsilon_5+\varepsilon_3\varepsilon_5},\\ \text{for } \varepsilon_5>\varepsilon_{5,MF} \text{ and } \varepsilon_3<\varepsilon_{3,5,MF}\left(\varepsilon_5\right).\end{cases} \tag{2-51}$$

Function $\gamma(\varepsilon_3,\varepsilon_5)$ is shown in Fig. 2-14.

The combination of functions $\gamma_1(\varepsilon_3,\varepsilon_5)$ and $\gamma_2(\varepsilon_3,\varepsilon_5)$ is made along the curve $\varepsilon_{3,5,\gamma}$, that is shown in Fig. 2-15.

The combination curve $\varepsilon_{3,5,\gamma}$ is the same as $\varepsilon_{3,5,\max}(\varepsilon_5)$ at the left hand from the $\varepsilon_{5,MF}$ point. It is suggested to call the $\varepsilon_{3,5,\max}(\varepsilon_5)$ as Maximum Power Line. The waveform of voltage v_C, that is appropriate to Maximum Power Line with $\varepsilon_3\approx-4.8308$, and $\varepsilon_5=25$ is shown in Fig. 2-16.

The combination curve $\varepsilon_{3,5,\gamma}$ is the same as $\varepsilon_{3,5,MF}(\varepsilon_5)$ at the right hand from the $\varepsilon_{5,MF}$ point. It is suggested to call the $\varepsilon_{3,5,MF}(\varepsilon_5)$ as Flatness Line. The waveform of voltage v_C, that is appropriate to Flatness Line with $\varepsilon_3=-6.75$, and $\varepsilon_5=75$ is shown in Fig. 2-17.

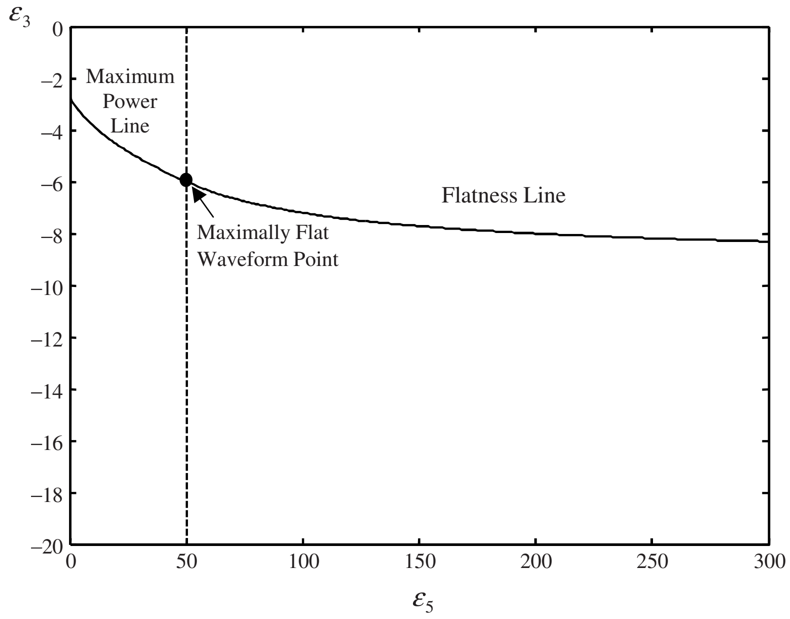

Figure 2-15. Curve $\varepsilon_{3,5,\gamma}$.

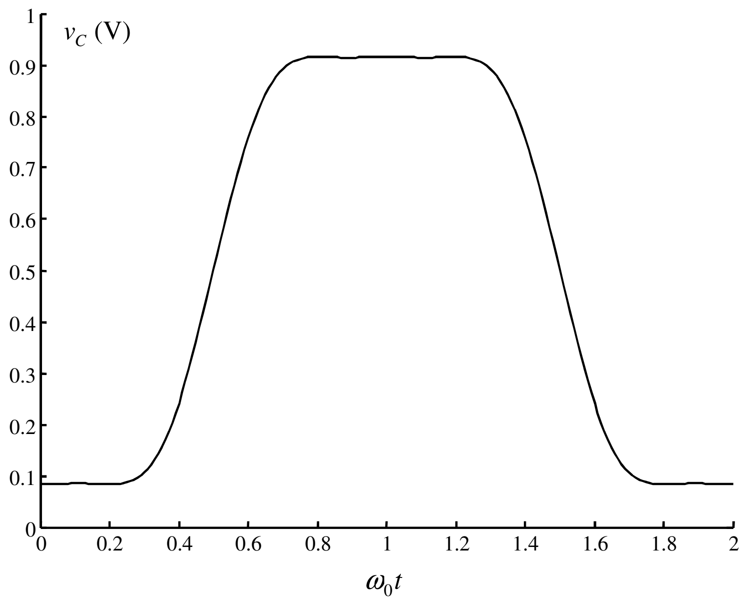

Figure 2-16. The waveform of voltage v_C , that is appropriate to Maximum Power Line with $\varepsilon_3 \approx -4.8308$, and $\varepsilon_5 = 25$.

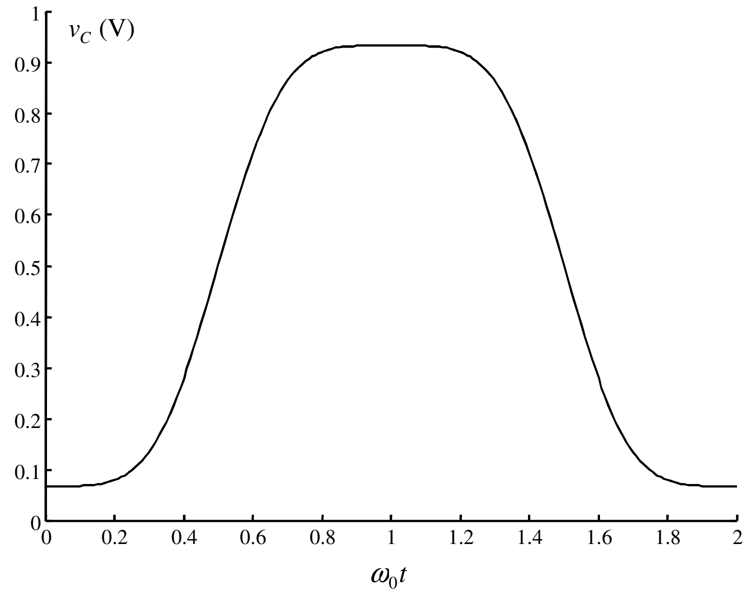

Figure 2-17. The waveform of voltage v_C, that is appropriate to Flatness Line with $\varepsilon_3 = -6.75$, and $\varepsilon_5 = 75$.

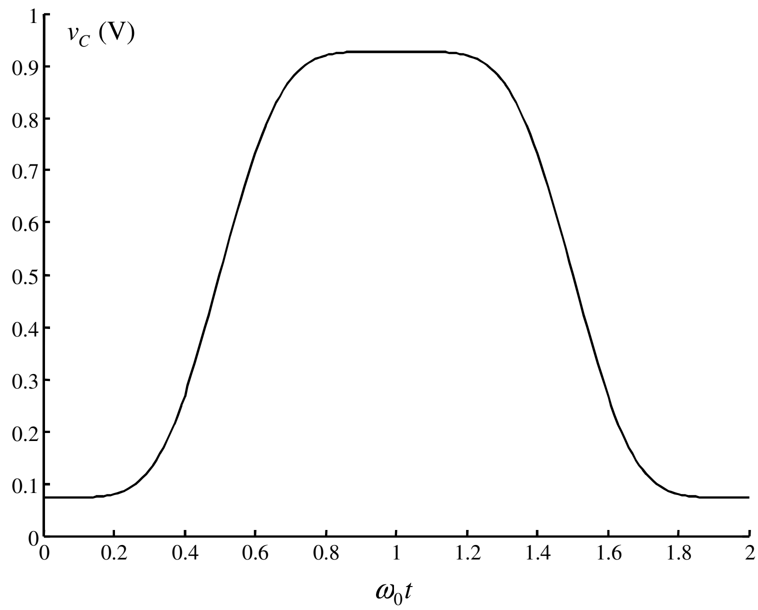

Figure 2-18. The maximally flat waveform of voltage v_C, that is appropriate to $\varepsilon_3 = -6$, and $\varepsilon_5 = \varepsilon_{5,MF} = 50$.

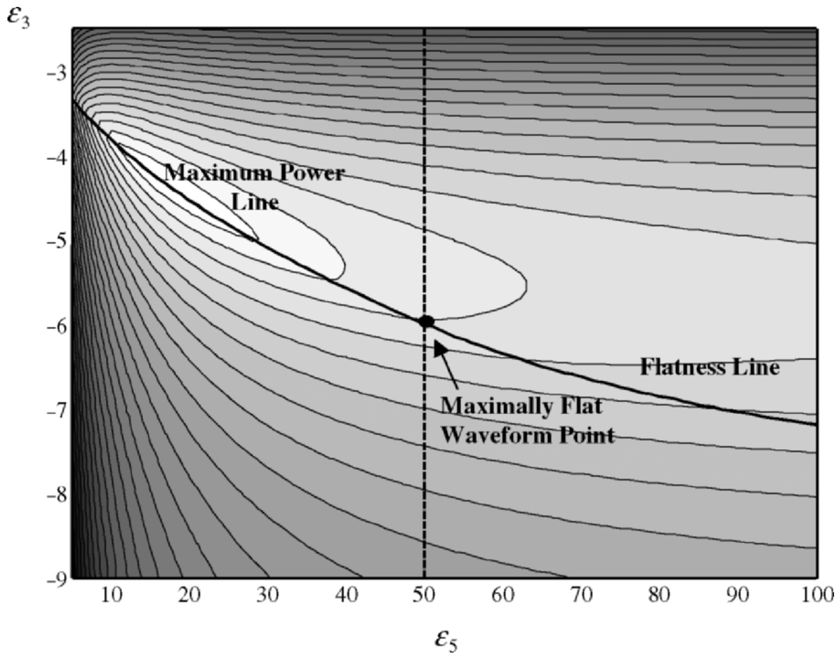

Figure 2-19. The contour plot of the function $\gamma(\varepsilon_3, \varepsilon_5)$.

The point of contingence, that is defined by Eqs. (2-48) and (2-50), is appropriate to the maximally flat waveform of voltage v_C. This is in agreement with Raab's analysis[14]. The maximally flat waveform of voltage v_C, that is appropriate to $\varepsilon_3 = -6$, and $\varepsilon_5 = \varepsilon_{5,MF} = 50$, is shown in Fig. 2-18.

As it was mentioned by Raab[27], the maximally flat waveform is not best of all in order to provide the maximum output power at the first harmonic frequency. In other words, the value of function $\gamma(\varepsilon_3, \varepsilon_5)$ is not maximal at the point of maximally flat waveform. The contour plot of the function $\gamma(\varepsilon_3, \varepsilon_5)$ is shown in Fig. 2-19.

It can be seen from Fig. 2-19, the maximal value of the function $\gamma(\varepsilon_3, \varepsilon_5)$ can be reached at the Maximum Power Line $\varepsilon_{3,5,\max}(\varepsilon_5)$. Really, the functions $\gamma_1(\varepsilon_3, \varepsilon_5)$ and $\gamma_2(\varepsilon_3, \varepsilon_5)$ have not local extremums. Therefore, they reach their maximal values at the boundary of region. The functions $\gamma_1(\varepsilon_3, \varepsilon_5)$ and $\gamma_2(\varepsilon_3, \varepsilon_5)$ are of the same values along the Maximum Power Line $\varepsilon_{3,5,\max}(\varepsilon_5)$. So, it is sufficient to find the conditional extremum of only one of them. In order to simplify the analysis, the function $\gamma_1(\varepsilon_3, \varepsilon_5)$ was selected. The derivative of function $\gamma_1(\varepsilon_3, \varepsilon_5)$ along the Maximum Power Line $\varepsilon_{3,5,\max}(\varepsilon_5)$ can be written as follows:

$$\frac{d\gamma_1\left(\varepsilon_{3,5,\max}\left(\varepsilon_5\right),\varepsilon_5\right)}{d\varepsilon_5}$$
$$=\frac{d}{d\varepsilon_5}\left(\frac{\varepsilon_{3,5,\max}\left(\varepsilon_5\right)\cdot\varepsilon_5}{\varepsilon_{3,5,\max}\left(\varepsilon_5\right)+\varepsilon_5+\varepsilon_{3,5,\max}\left(\varepsilon_5\right)\cdot\varepsilon_5}\right) \quad (2\text{-}52)$$
$$=\frac{\frac{d\varepsilon_{3,5,\max}\left(\varepsilon_5\right)}{d\varepsilon_5}\cdot\varepsilon_5^2+\varepsilon_{3,5,\max}^2\left(\varepsilon_5\right)}{\left(\varepsilon_{3,5,\max}\left(\varepsilon_5\right)+\varepsilon_5+\varepsilon_{3,5,\max}\left(\varepsilon_5\right)\cdot\varepsilon_5\right)^2}.$$

In view of substantial crockness, the conclusive expression for the derivative $d\gamma_1(\varepsilon_{3,5,\max}(\varepsilon_5),\varepsilon_5)/d\varepsilon_5$ is not provided. However, the derivative $d\gamma_1(\varepsilon_{3,5,\max}(\varepsilon_5),\varepsilon_5)/d\varepsilon_5$ at the $0<\varepsilon_5<50$ region is shown in Fig. 2-20.

The derivative $d\gamma_1(\varepsilon_{3,5,\max}(\varepsilon_5),\varepsilon_5)/d\varepsilon_5$ reaches its zero value at the $\varepsilon_{5,\max}\approx 16.48528$ point.

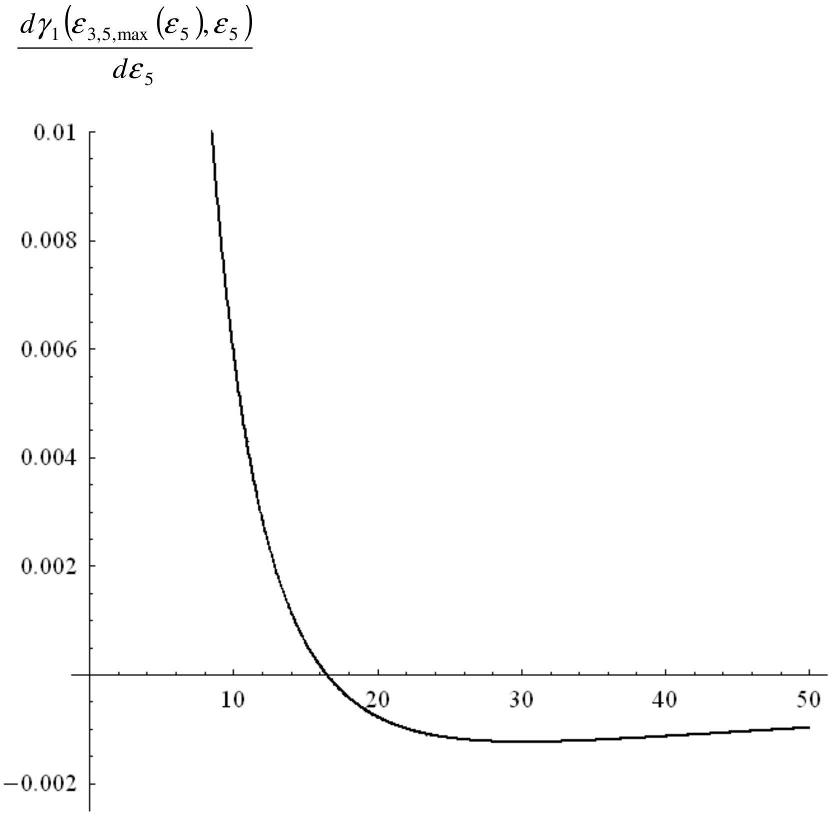

Figure 2-20. The derivative $d\gamma_1(\varepsilon_{3,5,\max}(\varepsilon_5),\varepsilon_5)/d\varepsilon_5$ at the $0<\varepsilon_5<50$ region.

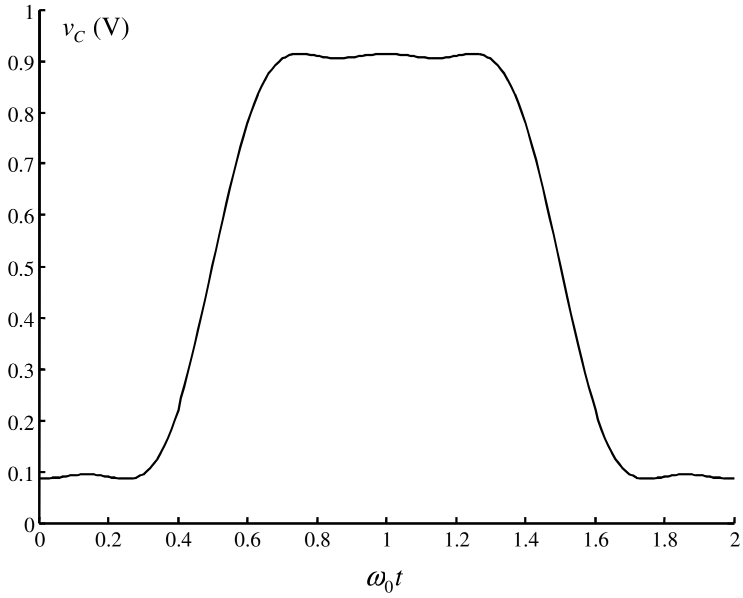

Figure 2-21. The waveform of voltage v_C, that is appropriate to maximum output power point $\varepsilon_3 = \varepsilon_{3,5,\max}(\varepsilon_{5,\max}) = -4.30602$, and $\varepsilon_5 = \varepsilon_{5,\max} = 16.48528$.

The ordinate $\varepsilon_{3,5,\max}(\varepsilon_{5,\max}) = -4.30602$ is appropriate to the absciss $\varepsilon_{5,\max}$. This is in agreement with Raab's analysis[27]. The corresponding maximum value of function $\gamma(\varepsilon_3, \varepsilon_5)$ is the following:

$$\gamma(\varepsilon_{3,5,\max}, \varepsilon_{5,\max}) = 1.207107 \quad (2\text{-}53)$$

The waveform of voltage v_C, that is appropriate to maximum output power point $\varepsilon_3 = \varepsilon_{3,5,\max}(\varepsilon_{5,\max}) = -4.30602$, and $\varepsilon_5 = \varepsilon_{5,\max} = 16.48528$, is shown in Fig. 2-21.

5. SUMMARY

The analysis of collector current spectrum changes with frequency growing is conducted using the approximated charge storage model.

The following result are achieved:

- The effect of collector current stretching at the higher frequencies leads to the additional phase shift between the collector-emitter voltage harmonics. It needs to be compensated for in order to achieve high efficiency.

- The general condition defining the boundaries of possible class-F realization for the different transistors is formulated by Eq. (2-26).

- The third harmonic input impedance of output network that compensates for the additional phase shift is given by Eqs. (2-27), and (2-28).

Therefore, the obtained results allow accounting the transistor lag and getting the optimal efficiency and output power characteristics at relatively high frequencies.